Conclusions

This Note presents a numerical scheme based on an integral approach, which is extended and modified to present sub-critical integral equation method and the wake model to transonic oscillatory solid and porous aerofoils in pitching motion. This method is able to capture reasonably sharp shocks spread over the aerofoil surface, whose strengths and locations agree well with the experimental data. The present approach shows that the use of a compressible linear operator with the addition of artificial viscosity to the governing equation is sufficient to produce convergence for the cases arising in inviscid isentropic flows with shocks.

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Nonlinear Static and Dynamic Analysis Method of Cable Structures

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Introduction

CABLES are very promising structural elements for space applications since they can be packaged easily in a small volume and function as truss members when in a stretched state.[1] However, an exact analysis of cable structures is difficult because of cable slackening and large deformations. This is particularly true when the cables form a network. Usually cables are modeled as pin-supported rods, and a linear, finite element analysis is employed.[2] If all the cables are in tension and the number of cables is greater than or equal to the degrees of freedom of the structure, a linear, finite element analysis gives a good approximation. However, if any cables are slack and/or the structure is a tension stabilized structure, an iterative technique has to be employed in a linear analysis. Herbert and Bachtell[3] reported a nonlinear analysis method that includes second-order strains and the effect of tension stabilization, but higher-order strains and the effect of cable slackening were neglected.

In this Note, a nonlinear cable structure analysis method is derived based on Hamilton's principle and a general purpose cable structure analysis code (CASA) is developed. Higher-order strains are included in the method by taking positions, not displacements, as variables, and cable slackenings are given by a nonlinear, axial stiffness of cable. Nonlinear static and linear dynamic analyses of structures composed of cables and/or trusses can be treated by the present method.

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Theory

Assume that the number of elements (cables or trusses) in the structure is C and the number of nodes is n. The strain energy in the structure is expressed by the summation of the strain energy in each element

$$P = \sum_{i=1}^{c} \frac{1}{2}\left(\frac{|\bar{X}_{i1} - \bar{X}_{i2}| - l_{0i}}{l_{0i}}\right)^2 (EA)_i l_{0i} \tag{1}$$

where l_{0i} denotes the initial length (length in nonstressed state) of ith element and $(EA)_i$ denotes the axial stiffness (product of Young's modulus and cross-sectional area) of ith element. $\bar{X}_{i1}$ and $\bar{X}_{i2}$ denote the position vectors of two terminal nodes of ith element. The axial stiffness $(EA)_i$ of a cable element is set to zero when the strain is negative. The strain energy variation corresponding to the node position variations is given as

$$\delta P = \sum_{i=1}^{c} \frac{(EA)_i(|\bar{X}_{i1} - \bar{X}_{i2}| - l_{0i})}{l_{0i}|\bar{X}_{i1} - \bar{X}_{i2}|} (\bar{X}_{i1} - \bar{X}_{i2}) \cdot (\delta\bar{X}_{i1} - \delta\bar{X}_{i2}) \tag{2}$$

An external force of magnitude f_j is imposed on the jth node from a fixed node, whose position vector is $\bar{X}_{fj}$. The virtual work is given as

$$\delta W = \sum_{j=1}^{n} f_j \frac{(\bar{X}_{fj} - \bar{X}_j)}{|\bar{X}_{fj} - \bar{X}_j|} \delta\bar{X}_j \tag{3}$$

Combining the variation of potential energy and the virtual work, according to the principle of stationary value of the total potential energy, yields

$$\bar{F}_1 \cdot \delta\bar{X}_1 + \bar{F}_2 \cdot \delta\bar{X}_2 + \ldots \bar{F}_n \cdot \delta\bar{X}_n = 0 \tag{4}$$

where $\bar{F}_i$ is a nonlinear function of vectors $\bar{X}_i$. When the number of fixed nodes is n_f, the nonlinear equations that describe the static equilibrium state are given as follows:

$$\begin{cases} \bar{F}_1(\bar{X}_1, \bar{X}_2, \ldots \bar{X}_m) = 0 \\ \vdots \\ \bar{F}_m(\bar{X}_1, \bar{X}_2, \ldots \bar{X}_m) = 0 \end{cases} \tag{5}$$

where $m = n - n_f$. Solving Eq. (5) using the Newton-Raphson method yields the static equilibrium state of the structure. The

Table 1 Displacement of node D and cable tensions corresponding to node E position

Node E X, Y position, Z = −1000 mm	Node D displacement, mm				Cable tensions, kgf			
	Present method		FEM		Present method		FEM	
	X, Y	Z	X, Y	Z	Cable 1	Cable 2,3	Cable 1	Cable 2,3
250 mm	−0.23	−0.81	−0.23	−0.81	0.61	0.47	0.61	0.47
500 mm	0.48	−0.48	0.48	−0.48	1.00	0.01	1.00	0.00
550 mm	16.19	16.01	0.57	−0.41	1.00	Slack	1.02	−0.07
750 mm	65.46	79.38	0.81	−0.21	1.00	Slack	1.01	−0.27

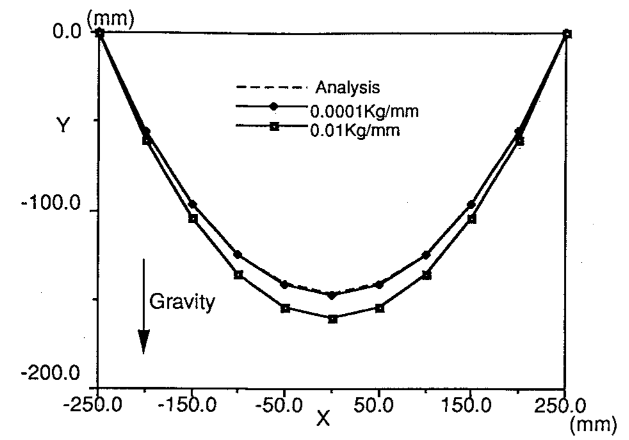

Fig. 1 Catenary curves calculated by present method and analysis.

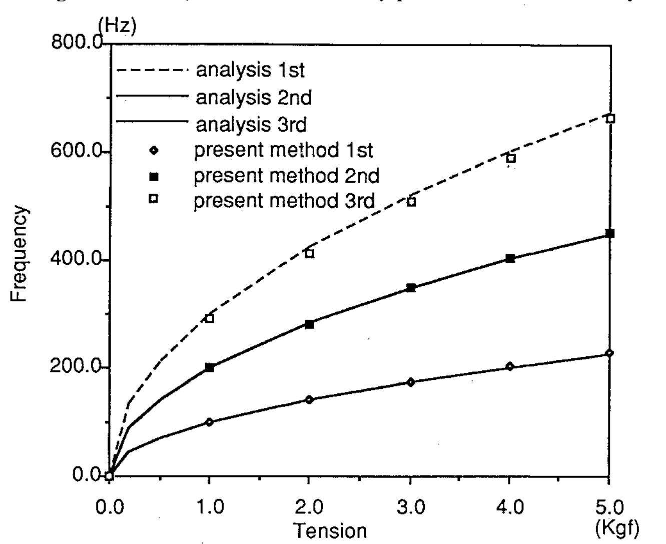

Fig. 2 Cable natural frequencies calculated by present method and analysis.

iterative scheme uses the Jacobian matrix in the form

$$\begin{Bmatrix} \bar{X}_1 \\ \vdots \\ \bar{X}_m \end{Bmatrix}^{K+1} = \begin{Bmatrix} \bar{X}_1 \\ \vdots \\ \bar{X}_m \end{Bmatrix}^{K} - \begin{bmatrix} \dfrac{\partial \bar{F}_1}{\partial \bar{X}_1} & \cdots & \dfrac{\partial \bar{F}_1}{\partial \bar{X}_m} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial \bar{F}_m}{\partial \bar{X}_1} & \cdots & \dfrac{\partial \bar{F}_m}{\partial \bar{X}_m} \end{bmatrix} \begin{Bmatrix} \bar{F}_1 \\ \vdots \\ \bar{F}_m \end{Bmatrix} \tag{6}$$

where the superscript K denotes the Kth estimation of node-position vectors.

The variation of the kinetic energy is expressed as

$$\delta T = -\sum_{i=1}^{n} m_i \ddot{\bar{X}}_i \cdot \delta \bar{X}_i \tag{7}$$

where m_i denotes the mass of the ith node. The equations of motion are obtained by combining Eqs. (5) and (7) in accordance with d'Alembert's principle:

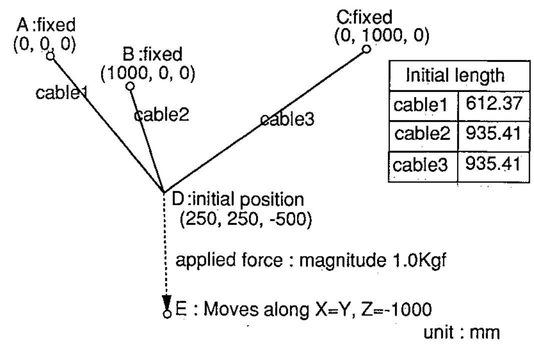

Fig. 3 Calculation model.

$$\begin{cases} \bar{F}_1(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_m) + m_1 \ddot{\bar{X}}_1 = 0 \\ \vdots \\ \bar{F}_m(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_m) + m_m \ddot{\bar{X}}_m = 0 \end{cases} \tag{8}$$

Differentiating the above equations with $\bar{X}_i$ about the static equilibrium state gives the linear vibration equation

$$\begin{bmatrix} m_1 & & 0 \\ & \ddots & \\ 0 & & m_m \end{bmatrix} \begin{Bmatrix} d\ddot{\bar{X}}_1 \\ \vdots \\ d\ddot{\bar{X}}_m \end{Bmatrix} + \begin{bmatrix} \dfrac{\partial \bar{F}_1}{\partial \bar{X}_1} & \cdots & \dfrac{\partial \bar{F}_1}{\partial \bar{X}_m} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial \bar{F}_m}{\partial \bar{X}_1} & \cdots & \dfrac{\partial \bar{F}_m}{\partial \bar{X}_m} \end{bmatrix} \begin{Bmatrix} d\bar{X}_1 \\ \vdots \\ d\bar{X}_m \end{Bmatrix} = \{0\} \tag{9}$$

Note that the stiffness matrix is identical to the Jacobian matrix used in the Newton-Raphson method in the static analysis. Natural frequencies and vibration modes can be calculated by a usual eigenvalue analysis.

Calculation Examples

Catenary

The catenary curve of a cable calculated using the present method is compared with an analytical solution as shown in Fig. 1. Cable axial stiffness and length are 100 Kgf and 600 mm, respectively. Two cable mass properties (0.0001 kg/mm and 0.01 kg/mm) were chosen to represent a light and a heavy cable. Calculations based on the present method used a model that divides the cable into ten cable elements. The result on the light cable agrees with the analytical solution, whereas that on the heavy cable shows larger deformation due to the axial elongation of the cable.

Vibrating Cable

Figure 2 shows the natural frequencies of a cable calculated by the present method and a linear vibration theory. Cable density, cross-sectional area, length of stretched cable, and Young's modulus are fixed to 0.0027 g/mm^3, 0.04 mm^2, 1500 mm, and E = 2500 kgf/mm^2, respectively. Numerical calculation based on the present method used a model that divided the cable into ten equal-length cable elements. It is shown that the present method gives a good approximation to the linear vibration theory.

Cable Network

Displacements and cable tensions of a statically determinant cable network shown in Fig. 3 are calculated by the present method and a linear finite element method (FEM) and results are compared. The axial stiffness of the cables is 785.5 kgf. A force of 1 kgf is imposed on node D from node E, which moves along the line of $X = Y \geq 0$, $Z = -1000$ mm. Displacements of node D and cable tensions corresponding to the node E position are shown in Table 1. When all cables are in tension ($X = Y \leq 500$), the results of the present method and FEM coincide. But when there is a slack cable ($X = Y > 500$), the model loses the statically determinant condition, and the linear FEM analysis results in a physically meaningless solution; cables 2 and 3 are subjected to compressive forces. Even in this case, the present method yields a mechanically balanced configuration of the network, in which cable 1 has a tension of 1.0 kgf, equivalent to the magnitude of the applied force, and is directed toward node E.

Conclusion

This Note outlines a numerical analysis method applicable to general cable structures and presents three sample calculations. The present method can be applied to analyses of cable networks that may cause cable slackening; mechanical deformations by means of taking node positions, not displacements, as variables; considering full-order strains of cables; and employing a nonlinear cable stiffness.

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Closed Form Solution for Minimum Weight Design with a Frequency Constraint

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Introduction

THE minimum weight design of structures with natural frequency constraints has been an active topic of research since the earlier work of Turner.[1] Recently, this problem has been re-examined by several researchers using approaches that combine finite-element analysis and numerical optimization techniques (see, e.g., Refs. 2–6). In this Note, closed form minimum weight design formulas are derived for one-dimensional discrete vibration systems (i.e., for systems with one degree of freedom per node) with one end fixed and the other end free. In practical application, shear building and torsional vibration of shafts can be represented as first approximation by such models. Our objective is to provide a closed form solution for optimum preliminary sizing for a specified natural frequency. Additionally, the results of the present Note can be used to generate test cases to verify the frequency constraint capability of general-purpose finite-element codes. Our results are derived based on the Lagrange multiplier techniques. It is very interesting to note that the results show that the optimal mode shape corresponding to the optimal design depends only on the specified natural frequency and the coefficients of the objective function. Using the optimal mode shape and the specified eigenvalue, the design parameter can then be computed from the eigenvalue problem. In the following sections, we will first state the design problems, and the main results will then be summarized in theorems. A numerical example is included to illustrate the results.

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Problem Statements

Consider the system shown in Fig. 1. The problem is to find minimum weight design for specified fundamental frequency of the system. This is problem A in which we ignore the spring mass. Mathematically, the problem can be stated as the following: Find spring constants x_1 to x_n to minimize

$$W = \sum_{i=1}^{n} c_i x_i$$

subject to the constraint $\lambda_1 = p$ where c_i are prescribed constants, λ_1 is the fundamental eigenvalue, and p is a prescribed positive value. If the effect of structural mass is included, the optimal problem can be stated the same as the preceding if we define $c_i = \rho_i L_i^2 / E_i$ and $x_i = E_i A_i / L_i$ where ρ_i, L_i, E_i are mass density, length, and Young's modulus, respectively, for element i (see Fig. 2). This is referred to as problem B in the Note.

In these problems, the eigenvalue λ_1 is an implicit function of the design variables. They are related through the eigenvalue problem

$$[K]\{\phi\} = \lambda_1 [M]\{\phi\} \tag{1}$$

where λ_1 is the fundamental eigenvalue, $\{\phi\}$ the corresponding mode shape, $[K]$ and $[M]$ the system stiffness and mass matrices, respectively, and where

$$[K] = [K(x)] \tag{2}$$

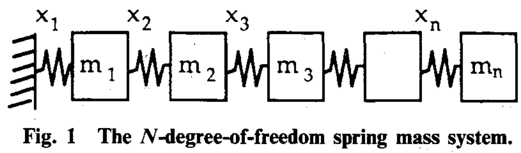

Fig. 1 The *N*-degree-of-freedom spring mass system.

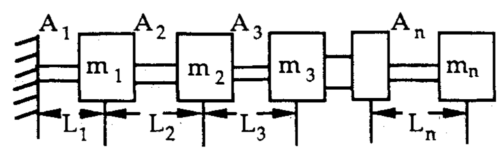

Fig. 2 The *N*-degree-of-freedom axial vibration system.